Resonance effects due to the excitation of surface Josephson plasma waves in layered superconductors

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Abstract

We analytically examine the excitation of surface Josephson plasma waves (SJPWs) in periodically-modulated layered superconductors. We show that the absorption of the incident electromagnetic wave can be substantially increased, for certain incident angles, due to the resonance excitation of SJPWs. The absorption increase is accompanied by the decrease of the specular reflection. Moreover, we find the physical conditions guaranteeing the total absorption (and total suppression of the specular reflection). These conditions can be realized for Bi2212 superconductor films.

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I. INTRODUCTION

Over the last twenty years, the physical properties of layered superconductors have attracted the attention of many research groups . The strongly-anisotropic high-temperature $Bi_2Sr_2CaCu_2O_{8+\delta}$ single crystals are the most prominent members of this family. Numerous experiments on the **c**-axis transport in layered high-T_c superconductors (HTS) justify the use of a model in which the superconducting CuO_2 layers are coupled, through the block layers, by the intrinsic Josephson effect. The Josephson current flowing along the **c**-axis is coupled with the electromagnetic field inside the insulating dielectric layers, thereby causing a specific kind of elementary excitations called Josephson plasma waves (JPWs) (see, e.g., Ref. 1). In other words, the layered structure of Bi-based superconductors and related compounds favors the propagation of electromagnetic waves through the layers. These waves are of considerable interest because of their terahertz (THz) and sub-THz frequency ranges, which are still hardly reachable for both electronic and optical devices. The frequencies of terahertz waves are in the region of resonance frequencies of molecules and are expected to have many applications.

The unusual optical properties of layered superconductors, including their reflectivity and transmissivity, caused by the JPWs excitation, were studied, e.g., in Ref. 2. Earlier works on this problem have focused on the propagation of bulk waves, that is possible in the frequency range above the Josephson plasma frequency ω_J, at $\omega > \omega_J$, only. The presence of the sample boundary can produce a new branch of the wave spectrum below the Josephson plasma frequency, $\omega < \omega_J$, i.e., surface Josephson plasma waves (SJPWs), which are an analog of the surface plasmon polaritons[3,4]. Recently, the existence of SJPWs in layered superconductors in the THz frequency range was predicted[5,6]. Surface waves play an important role in many fundamental resonance optics phenomena[7], such as the Wood's anomalies in reflectivity[4,8] and transmissivity[9,10] of periodically-corrugated metal samples. A recent overview of unusual resonators can be found in Ref. 7. Therefore, it is essential to study similar resonance phenomena caused by the excitation of surface waves in layered superconductors.

The dispersion curve, $\omega(q)$, of the surface waves lies below the "vacuum light line", $\omega = cq$, where q is the wave-number and c is the speed of light. This means that the surface waves have wave-vectors greater than the wave-vectors of light of the same frequency in

vacuum. Thus, to excite the surface waves by means of incident irradiation, it is necessary to use special methods[4], such as, e.g., the attenuated total reflection (ATR) method and the surface modulation method.

In this paper, we study the excitation of surface Josephson plasma waves while diffracting the electromagnetic wave incident onto the periodically-modulated layered superconductor. For simplicity, we present results for single-resonance cases, when only one SJPW is excited. The excitation of SJPWs affects the absorption and reflection of the incident electromagnetic waves, specifically, determining their resonance dependence on the frequency ω and the incident angle θ. These phenomena are potentially useful for detecting THz radiation.

II. MODEL

Consider a semi-infinite layered superconductor in the simplest geometry shown in Fig. 1. The crystallographic **ab**-plane coincides with the xy-plane and the **c**-axis is directed along the z-axis. Superconducting layers are numbered by an integer $l \geq 1$.

Suppose that the maximum **c**-axis Josephson current density, J_c, is periodically modulated in the x-direction with a spatial period L. Such a modulation can be realized, for instance, either by irradiating a standard $Bi_2Sr_2CaCu_2O_{8+\delta}$ sample covered by a modulated mask[11] or by pancake vortices controlled by an out-of-plane magnetic field[12].

A p-polarized (transverse magnetic) plane monochromatic electromagnetic wave with electric, $\mathbf{E} = \{E_x, 0, E_z\}$, and magnetic, $\mathbf{H} = \{0, H, 0\}$, fields is incident onto a periodically-modulated layered superconductor at an angle θ from the vacuum half-space. The in-plane and out-of-plane components of its wave-vector are

$$k_x \equiv q = k \sin\theta, \quad k_z = -k\cos\theta, \quad k = \omega/c.$$

The in-plane periodic modulation results in generating the diffracted waves with in-plane and out-of-plane wave-vector components,

$$q_n = q + ng, \quad k_{zn}^V = \sqrt{k^2 - q_n^2}, \quad \mathrm{Re}[k_{zn}^V],\ \mathrm{Im}[k_{zn}^V] \geq 0,$$

n is an integer and $g = 2\pi/L$. The resonance excitation of the SJPWs corresponds to the condition,

$$q_n = k\sin\theta + ng = \mathrm{sign}(n)\mathrm{Re}[\kappa_{\mathrm{sw}}(\omega)], \tag{1}$$

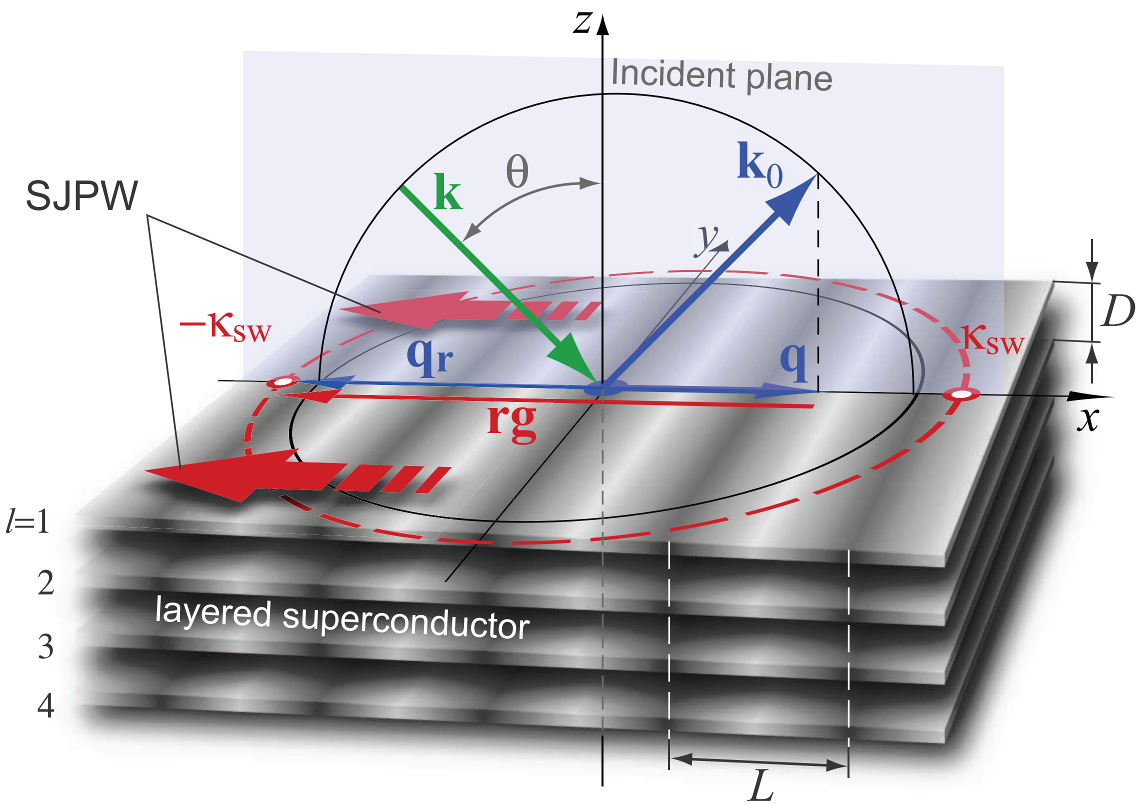

FIG. 1: (Color online) Geometry of the problem: $\mathbf{k}$ and $\mathbf{k_0}$ are the wave-vectors of the incident and specularly reflected waves, $\kappa_{\rm sw}$ is the wave-number of the SJPW. The case of backward resonance diffraction in the rth order ($r < 0$) is shown. Also, $q_r = q + rg \simeq -\kappa_{\rm sw}$ denotes the tangential component of the wave-vector of the resonance wave, and $g = 2\pi/L$ is the period of the reciprocal grating.

where $\kappa_{\rm sw}(\omega) > k$ is the SJPW wave-number[6],

$$\kappa_{\rm sw}^2(\omega) = k^2 \left[1 - \frac{k^2 \lambda_{ab}^2 \Omega^2}{\varepsilon(1-\Omega^2 - i\nu\Omega)}\right]^{-1}. \tag{2}$$

The total magnetic field in the vacuum ($z > 0$) is given by the Fourier-Floquet expansion,

$$\begin{aligned} H^V(x,z) = H^{\rm inc}\Big[&\exp(iqx - ikz\cos\theta) + \\ &\sum_n R_n \exp(iq_n x + ik_{zn}^V z)\Big], \end{aligned} \tag{3}$$

where $H^{\rm inc}$ is the amplitude of the incident wave and R_n are the transformation coefficients (TCs). The time dependence $\exp(-i\omega t)$ is omitted hereafter.

Using Maxwell equations, we express the tangential component of the electric field in the vacuum in terms of the magnetic field,

$$\begin{aligned} E_x^V(x,z) = H^{\rm inc}\Big[&-\beta^V \exp(iqx - ikz\cos\theta) \\ &+ \sum_n \beta_n^V R_n \exp(iq_n x + ik_{zn}^V z)\Big], \end{aligned} \tag{4}$$

where $\beta^V = \cos\theta$, $\beta_n^V = k_{zn}^V/k$.

The electromagnetic field within the layered superconductor is related to the gauge-invariant phase difference, φ_l, of the order parameter. The values of φ_l in the junctions are governed by the set of coupled sine-Gordon equations (see, e.g., Ref. 13),

$$\begin{aligned} \left(1 - \frac{\lambda_{ab}^2}{D^2}\partial_l^2\right) &\left(\frac{\partial^2 \varphi_l}{\partial t^2} + \omega_r \frac{\partial \varphi_l}{\partial t} + \omega_{\scriptscriptstyle J}^2(x)\sin\varphi_l\right) \\ &- \frac{c^2}{\varepsilon}\frac{\partial^2 \varphi_l}{\partial x^2} = 0. \end{aligned} \tag{5}$$

Here λ_{ab} is the London penetration depth across the layers, D is the spatial period of the layered structure, the discrete second derivative operator ∂_l^2 is defined as $\partial_l^2 f_l = f_{l+1} + f_{l-1} - 2f_l$,

$$\omega_r = \frac{4\pi\sigma_c}{\varepsilon}$$

is the relaxation frequency, σ_c is the quasi-particle conductivity across the layers,

$$\omega_{\scriptscriptstyle J}(x) = \sqrt{\frac{8\pi e D J_c(x)}{\hbar\varepsilon}} \tag{6}$$

is the periodically-modulated Josephson plasma frequency, and ε is the interlayer dielectric constant. The Fourier expansion of $\omega_{\scriptscriptstyle J}^2(x)$ is

$$\omega_{\scriptscriptstyle J}^2(x) = \omega_{\scriptscriptstyle J}^2\left[1 + \sum_{n\neq 0} f_n \exp(ingx)\right], \quad f_{-n} = f_n^*. \tag{7}$$

Below we assume the modulation to be small, $|f_n| \ll 1$.

As was shown in Ref. 14, the intralayer quasi-particle conductivity, σ_{ab}, should also be taken into account if ω is far enough from the Josephson plasma frequency. The contribution of the in-plane conductivity to the dissipation can be easily incorporated in our analysis. However, for the frequency range considered here (close to ω_J), this contribution is strongly suppressed and can be safely omitted because the relative value of the term with σ_{ab} is

$$\left(\frac{\lambda_{ab}}{\lambda_c}\right)^2 \left(\frac{\sigma_{ab}}{\sigma_c}\right) \left|1 - \frac{\omega}{\omega_J}\right| \ll 1.$$

Here $\lambda_c = c/(\omega_J \sqrt{\varepsilon})$ is the London penetration depth along the layers.

For Josephson plasma waves, the nonlinear equations (5) can be linearized, i.e., $\sin\varphi_l$ can be replaced by φ_l. We also assume that the gauge-invariant phase difference experiences small changes, $|\varphi_{l+1} - \varphi_l| \ll |\varphi_l|$, and thus we can use the continuum approach, replacing $D^{-1}\partial_l \varphi_l$ by $\partial_z \varphi(z)$. Then Eq. (5) yields

$$\left(1 - \lambda_{ab}^2 \frac{\partial^2}{\partial z^2}\right) \left(\omega_J^2(x) - \omega^2 - i\omega\omega_r\right) \varphi - \frac{c^2}{\varepsilon} \frac{\partial^2 \varphi}{\partial x^2} = 0. \tag{8}$$

The magnetic and electric fields are related to the gauge invariant phase difference as

$$\frac{\partial \varphi}{\partial x} = \frac{2\pi D}{\Phi_0} \left(1 - \lambda_{ab}^2 \frac{\partial^2}{\partial z^2}\right) H, \tag{9}$$

$$E_x = -ik\lambda_{ab}^2 \frac{\partial H}{\partial z}, \quad E_z = ik \frac{\Phi_0}{2\pi D} \varphi \tag{10}$$

where $\Phi_0 = \pi c\hbar/e$ is the flux quantum and e is the elementary charge.

III. DIFFRACTION OF THE ELECTROMAGNETIC FIELD

Inside the layered superconductor, we represent the gauge-invariant phase difference and the electromagnetic field as expansions over the eigenfunctions,

$$\varphi(x, z) = H^{\rm inc} \sum_s \bar{C}_s \bar{\Psi}_s(x) \exp(p_s z), \tag{11}$$

$$H(x, z) = H^{\rm inc} \sum_s C_s \Psi_s(x) \exp(p_s z), \tag{12}$$

$$E_x(x, z) = -H^{\rm inc} \sum_s a_s C_s \Psi_s(x) \exp(p_s z), \tag{13}$$

with

$$\bar{\Psi}_s(x) = \sum_n \bar{\Psi}_{s|n} \exp(iq_n x), \;\; \Psi_s(x) = \sum_n \Psi_{s|n} \exp(iq_n x). \tag{14}$$

Here we introduce the dimensionless variable

$$a_s = -ik\lambda_{ab}^2 p_s.$$

Substituting the expressions (11)–(14) in Eqs. (8)–(10) gives a set of linear equations which allows us to find the coefficients $\bar{\Psi}_{s|n}$, $\Psi_{s|n}$ in the expansions (14) and the eigen-numbers p_s. After excluding the coefficients $\bar{\Psi}_{s|n}$, we arrive at the set of equations for $\Psi_{s|n}$. It can be solved by perturbations with respect to the small modulation, $|f_n| \ll 1$. In linear approximation, and in the absence of the degeneracy of the corresponding matrix, i.e., at

$$q_s^2 \neq q_n^2 \quad \text{for} \quad s \neq n, \tag{15}$$

we obtain

$$\begin{gathered} \Psi_{s|n} = \delta_{s,n} + \tilde{\Psi}_{s|n}, \;\; \tilde{\Psi}_{s|n} \simeq \frac{q_s^2}{q_n^2 - q_s^2}\tau_{n-s}, \;\; s \neq n, \\ \tau_s = \frac{f_s}{1 - \Omega^2 - i\nu\,\Omega}, \quad |\tau_s| \ll 1; \end{gathered} \tag{16}$$

$$\begin{gathered} p_s \simeq \frac{1}{\lambda_{ab}}\sqrt{1 + \frac{\lambda_c^2 q_s^2}{1 - \Omega^2 - i\nu\Omega}} + O(|\tau|^2), \\ \mathrm{Re}[p_s] > 0, \quad \mathrm{Im}[p_s] > 0, \end{gathered} \tag{17}$$

where $\delta_{s,n}$ is the Kronecker delta, $\Omega = \omega/\omega_J$, $\nu = \omega_r/\omega_J$.

Matching the tangential components of the electric and magnetic fields at the interface $z = 0$, we obtain an infinite set of linear algebraic equations for the coefficients C_s and their relations to the TCs R_n:

$$\sum_s D_{n|s} C_s = 2\beta^V \delta_{n,0}, \tag{18}$$

$$R_n = \sum_s C_s \Psi_{s|n} - \delta_{n,0}, \tag{19}$$

where

$$D_{n|s} = b_n \delta_{n,s} + d_{n|s}, \quad b_n = \beta_n^V + a_n, \tag{20}$$

$$d_{n|s} = (\beta_n^V + a_s)\tilde{\Psi}_{s|n}. \tag{21}$$

To solve the infinite set of equations (18) for C_s we use resonant perturbation theory, which allows presenting results in an explicit analytical form[15].

When all spatial field harmonics are far away from the eigen-modes of the unmodulated layered superconductor (nonresonance conditions), the diagonal elements b_s of the matrix $\hat{D} \equiv \|D_{n|s}\|$ are of the order of one or larger, $|b_s| \sim |\beta_s^V| \gtrsim 1$. In this case, the matrix $\hat{D}$ is diagonal-dominated, that is, its off-diagonal elements are small compared to the diagonal ones, $|d_{n|s}| \sim |\tau_{n-s}| \ll |b_s|$. Then, the solution of Eqs. (18), (19) gives us a trivial result: the specular reflection TC, R_0, is close to the Fresnel coefficient,

$$R_F = \frac{\cos\theta - a_0}{\cos\theta + a_0} \equiv |R_F| \exp(i\psi), \tag{22}$$

and differs from it by terms proportional to τ^2. Other TCs are small, $R_n \sim \tau_n$, $n \neq 0$.

A much more interesting case occurs under the resonance conditions, when Eq. (1) holds for one (or simultaneously for two) spatial field (resonance) harmonics,

$$q_r = k \sin\theta + rg \simeq \mathrm{sign}(r)\mathrm{Re}[\kappa_{\mathrm{sw}}]. \tag{23}$$

Here $r > 0$ ($r < 0$) corresponds to the forward (backward) propagation of the excited SJPW with respect to the incident wave.

For simplicity, we restrict ourselves to the single-resonance case. In the resonant case, the diagonal matrix element $D_{r|r} = b_r$ becomes anomalously small, and the determinant of the matrix $\hat{D}$ decreases significantly (see, e.g., Ref. 15). Recall that the normalized z-component of the wave-vectors in vacuum, β_s^V, can be either purely real or purely imaginary. Therefore, the minimum of $|b_r| \ll 1$ holds in the vicinity of the point in the (ω, θ)-plane where $\mathrm{Im}[\beta_r^V] = -\mathrm{Im}\,[a_r]$, which is the dispersion relation for SJPWs, Eq. (2).

Thus, the set of equations Eq. (18) consists of one resonance equation (with $n = r$),

$$D_{r|r} B_r + \sum_{N \neq r} D_{r|N} B_N = 0, \tag{24}$$

and the subset of nonresonance equations (with "nonresonance numbers" $N \neq r$). Solving the subset for the nonresonance coefficients B_N we obtain

$$B_N = 2\beta^V (\hat{M}^{-1})_{N|0} - B_r \sum_{N'} (\hat{M}^{-1})_{N|N'} D_{N'|r}, \tag{25}$$

where $\hat{M}^{-1}$ is the matrix inverse of the nonresonance square submatrix $\hat{M} = \|D_{N|N'}\|$. Substituting B_N in Eq. (24) we obtain

$$B_r = \frac{F_r}{\tilde{D}_{r|r}}, \tag{26}$$

where

$$\tilde{D}_{r|r} = D_{r|r} - \sum_{N,N'} D_{r|N}(\hat{M}^{-1})_{N|N'} D_{N'|r}, \tag{27}$$

$$F_r = -2\beta^V \sum_N D_{r|N}(\hat{M}^{-1})_{N|0}. \tag{28}$$

We now examine the solution Eqs. (25), (26) in the main approximation, i.e., taking into account the linear-in-τ term in F_r,

$$F_r = -\frac{2\beta^V d_{r|0}}{b_0}, \tag{29}$$

and quadratic-in-τ terms in $\tilde{D}_{r|r}$,

$$\tilde{D}_{r|r} = \beta_r^V + a_r + C_r, \quad C_r = -\sum_N \frac{d_{r|N} d_{N|r}}{b_N}. \tag{30}$$

In this approximation we keep only the zero-order term in the series expansion of $\hat{M}^{-1} \simeq \|\delta_{N,N'}/b_N\|$. Thus, we obtain

$$B_r = \frac{F_r}{\beta_r^V + \xi\beta_r + C_r}, \tag{31}$$

$$B_N = \frac{2\beta^V \delta_{N,0} - d_{N|r} B_r}{b_N}. \tag{32}$$

Finally, using Eqs. (19), (16) we derive the resonance, R_r, and nonresonance, R_N, transformation coefficients,

$$R_r = B_r, \quad R_N = R_F \delta_{N,0} + R_r \left(\tilde{\Psi}_{r|N} - \frac{d_{N|r}}{b_N} \right). \tag{33}$$

It is convenient to present the resonance TC, R_r, in the form

$$R_r = \frac{F_r}{\beta_r^V + a_r + C_r(\theta, \Omega, \tau)}, \tag{34}$$

where $C_r \equiv C_r(\theta, \Omega, \tau)$ is the parameter that describes the *coupling* between waves in the vacuum and the layered superconductor. Below we assume the coupling parameter C_r to be small. However, even when $|C_r| \ll 1$, the coupling of the waves in the vacuum and superconductor plays a very important role in the excitation of SJPWs and in the anomalies of the reflection properties (Wood's anomalies).

First, the dispersion relation of the surface Josephson plasma waves is modified, involving the radiation leakage in the vacuum. The new spectrum of the SJPWs is defined by equating the denominator in Eq. (34) to zero. Thus, the quadratic in the modulation term, C_r, is

responsible for the shift of the position of the resonance, $\mathrm{Im}[C_r]$, and its widening, $\mathrm{Re}[C_r]$. The region where the coupling $|C_r| \ll 1$ (when the radiation leakage of the excited SJPW does not dominate) corresponds to the strongest excitation of the surface waves by the incident waves.

Second, due to the coupling, the specular reflection coefficient, R_0, in Eq. (33) differs from the Fresnel coefficient, R_F, and its modulus becomes less than one. Moreover, as we show below, the reflection of waves with any given frequency $\omega < \omega_J$ can be totally suppressed, for the specific incident angle θ and the modulation magnitude. This provides a way to control and filter the THz radiation.

In the next section, we study in detail the strong effects in the excitation of the SJPWs, the enhancement of absorptivity, and the suppression of the specular reflectivity near the resonance.

IV. SUPPRESSION OF THE SPECULAR REFLECTION

The transformation coefficient R_0 for the specularly-reflected wave, Eq. (33), can be rearranged as

$$R_0 = R_F \frac{k_{zr}^V/k + a_r + C_r(\theta,\Omega,\tau) - \Delta_r(\theta,\Omega,\tau)}{k_{zr}^V/k + a_r + C_r(\theta,\Omega,\tau)}, \tag{35}$$

where τ stands for τ_r, and

$$\Delta_r(\theta,\Omega,\tau) = \frac{2\cos\theta}{\cos^2\theta - a_0^2}\,(a_0 - a_r)\,\tilde{\Psi}_{0r}\tilde{\Psi}_{r0}. \tag{36}$$

To study the resonance phenomena, we consider the case most suitable for their observation, when the following inequalities are satisfied:

$$\frac{D^2}{\lambda_{ab}^2}\sin^2\theta \ll (1-\Omega^2)\varepsilon \ll 1. \tag{37}$$

The left inequality corresponds to the continuum limit for the field distribution in the z-direction, whereas the right inequality allows neglecting unity under the square root in Eq. (17). Besides, we assume the dissipation parameter ν to be small as compared to $(1-\Omega^2)$,

$$\nu \ll \left(1-\Omega^2\right). \tag{38}$$

For this frequency region, the complex parameter $a_r = a_r(\theta, \Omega) \equiv a'_r + i a''_r$ can be presented as

$$a_r = \frac{k^2 \lambda_{ab} \lambda_c}{2\sqrt{1-\Omega^2}} \left(\frac{\nu}{1-\Omega^2} - 2i \right) |\bar{q}_r|, \tag{39}$$

where we introduce the dimensionless variable

$$\bar{q}_n = \frac{q_n}{k} = \sin\theta + n\frac{g}{k}.$$

When restrictions Eqs. (37), (38) are valid, the expression for the reflectivity coefficient can be significantly simplified. First, the phase ψ of the Fresnel reflectivity coefficient, Eq. (22), is small,

$$\psi \simeq 2\frac{k^2 \lambda_{ab} \lambda_c}{\sqrt{1-\Omega^2}} \tan\theta \ll 1. \tag{40}$$

Second, the parameter a_r in Eq. (39) depends weakly on the angle θ in the vicinity of the resonance, whereas it depends strongly on the frequency detuning $(1-\Omega)$, and its real part is sensitive to the magnitude ν of the damping. Note also that near the resonance, $\Delta_r(\theta, \Omega, \tau)$ in Eq. (35) is almost real, $\Delta_r(\theta, \Omega, \tau) \simeq -2\mathrm{Re}\,[C_r(\theta, \Omega, \tau)]$.

Vanishing the imaginary part of the denominator in Eqs. (34), (35),

$$\mathrm{Im}[k_{zr}^V/k + a_r + C_r(\theta, \Omega, \tau)] = 0, \tag{41}$$

defines a curve in the (Ω, θ)-plane, where $|R_r(\Omega, \theta)|$ achieves its maximum. In view of assumed smallness of the coupling coefficient C_r, this curve passes close to

$$\theta = \theta_0 \equiv \arcsin \left| 1 - r\frac{g}{k} \right|. \tag{42}$$

Separating the real and imaginary parts in the numerator and denominator in Eq. (35), we rewrite the specular reflection coefficient R_0 the form,

$$R_0 = \frac{X_r(\vartheta, \Omega) + i\Big[\mathrm{Re}[C_r(\Omega, \tau)] - C_{\mathrm{opt}}(\Omega)\Big]}{X_r(\vartheta, \Omega) - i\Big[\mathrm{Re}[C_r(\Omega, \tau)] + C_{\mathrm{opt}}(\Omega)\Big]}, \tag{43}$$

where we introduce the incident-angle deviation $\vartheta = \theta - \theta_0$. For simplicity, below we restrict ourselves to the case of harmonic modulation and consider the resonances in the plus- and minus-first orders, $r = \pm 1$. Then,

$$X_r(\vartheta, \Omega) \simeq r\cos\theta_0 \frac{1-\Omega^2}{k^4 \lambda_{ab}^2 \lambda_c^2} \cdot (\vartheta - \vartheta_{\mathrm{res}}), \tag{44}$$

$$\vartheta_{\rm res} \simeq 4\frac{2-g^2/k^2}{(4-g^2/k^2)^2}\frac{k^4\lambda_{ab}^2\lambda_c^2}{1-\Omega^2}\,\frac{|\tau_r|^2}{\cos\theta_0}, \tag{45}$$

$$\mathrm{Re}[C_r(\Omega,\tau)] \simeq \sqrt{\frac{k}{g}}\frac{(1-rg/k)^2}{(2-rg/k)^{5/2}}\,\frac{k^2\lambda_{ab}\lambda_c}{\sqrt{1-\Omega^2}}\,|\tau_r|^2, \tag{46}$$

$$C_{\rm opt}(\Omega) \simeq \frac{\nu}{2(1-\Omega^2)}. \tag{47}$$

Equations (43)–(47) show that the modulus of the specular reflectivity $R_0(\theta)$ has a sharp resonance minima at $\vartheta = \vartheta_{\rm res}$,

$$|R_0|_{\rm min} \simeq \frac{|C_{\rm opt}(\Omega) - \mathrm{Re}[C_r(\Omega,\tau)]|}{C_{\rm opt}(\Omega) + \mathrm{Re}[C_r(\Omega,\tau)]}. \tag{48}$$

Its angular width, $\delta\vartheta$, is

$$\delta\vartheta = \frac{\nu}{(1-\Omega^2)^2}\frac{k^4\lambda_{ab}^2\lambda_c^2}{\cos\theta_0} \ll 1. \tag{49}$$

It is clearly seen that $|R_0|_{\rm min}$ depends strongly on the frequency detuning $(1-\Omega)$, dissipation parameter ν, and the coupling between waves in the vacuum and the layered superconductor, i.e., on the modulation magnitude $|f_r|$. This offers several important applications of the predicted anomaly of the reflectivity in the THz range. For instance, if the coupling parameter $\mathrm{Re}[C_r(\Omega,\tau)]$ is equal to $C_{\rm opt}$, i.e., the modulation magnitude $|f_r|$ takes on the optimal value,

$$|f_r|^2_{\rm opt} \simeq \frac{\nu}{2}\sqrt{\frac{g}{k}}\frac{(2-rg/k)^{5/2}}{(1-rg/k)^2}\frac{(1-\Omega^2)^{3/2}}{k^2\lambda_{ab}\lambda_c}, \tag{50}$$

then the specular reflection coefficient R_0 at $\vartheta = \vartheta_{\rm res}$ vanishes. This means that, by appropriate choice of the parameters, the total suppression of the reflectivity can be achieved due to the resonance excitation of the surface Josephson plasma wave.

In the vicinity of the resonance, the relative amplitude of the excited SJPW can be approximated by

$$R_r \simeq 2i\frac{\tau_r\cdot(1-\sin\theta_0)\tan^2\theta_0}{X_r(\vartheta,\Omega) - i\Big[\mathrm{Re}[C_r(\Omega,\tau)] + C_{\rm opt}(\Omega)\Big]}. \tag{51}$$

Note that equations $\vartheta = \vartheta_{\rm res}$ and $\mathrm{Re}[C_r(\Omega,\tau)] = C_{\rm opt}$ (i.e., $|f_r| = |f_r|_{\rm opt}$) constitute the conditions not only for the total suppression of the specular reflection, but also for the best matching of the incident wave and the SJPW. Under such conditions, the amplitude of the excited surface wave is much higher than the amplitude of the incident wave,

$$|R_r|_{\rm max} \simeq \frac{\sqrt{2}(1-\Omega^2)^{3/4}}{k\sqrt{\nu\lambda_{ab}\lambda_c}}\,\frac{\sin\theta_0}{\cos^{3/4}\theta_0} \gg 1. \tag{52}$$

Thus, we can achieve a high concentration of THz radiation energy in the SJPW.

The resonant decrease of the amplitude of the specularly-reflected wave is accompanied by the resonant increase of the absorption. Evidently, for the optimal conditions, $\vartheta = \vartheta_{\rm res}$, $|f_r| = |f_r|_{\rm opt}$, which correspond to the total suppression of the specular reflectivity, the energy pumped into the layered superconductor from the vacuum can be completely transformed into Joule heat due to the quasiparticle resistance. For the diffraction on the harmonic grating, the dependence of the absorptivity coefficient A on the wave frequency and the incident angle is described by a resonance curve,

$$\begin{aligned} A(\vartheta,\Omega) &= 1 - |R_0(\vartheta,\Omega)|^2 \\ &\simeq \frac{4C_{\rm opt}(\Omega){\rm Re}[C_r(\Omega,\tau)]}{X_r^2(\vartheta,\Omega) + \big[{\rm Re}[C_r(\Omega,\tau)] + C_{\rm opt}(\Omega)\big]^2}, \end{aligned} \tag{53}$$

accurate within terms of order $|\tau_r|^2$. It should be noted that the resonance increase of the electromagnetic absorption can result in a transition of the superconductor into normal state. Thus, new kinds of resonance phenomena can be observed in layered superconductors due to the excitation of the SJPWs. However, for rather low intensities of the incident wave, the sample heating due to the Joule losses can be neglected.

We have illustrated our analytical results by the numerical calculations of the specular and resonance TCs given in Eqs. (35), (34). The angular dependences of $|R_0|^2$ and $|R_r|^2$ for the forward resonance diffraction in the first diffraction order ($r = +1$) on a harmonic grating are shown in Fig. 2. The asymptotic formulae (43), (51) are in a good agreement with these plots. The modulation magnitude, $|f_r|$, was chosen to achieve the total suppression of the specular reflection. Its value is close to $|f_r|_{\rm opt}$ defined by the asymptotic expression (50).

The minimum in the specular reflectivity (see Fig. 2) is caused by the destructive interference of the waves scattered via two different channels. The first channel is the direct (approximately total) reflection from the unmodulated vacuum-layered superconductor interface. The magnetic-field amplitude of this wave is approximately equal to that of the incident wave, $R_F \simeq 1$. The second channel is defined by a two-step scattering process: the diffraction of the incident wave into the SJPW and the re-scattering of the SJPW into the specular direction. By means of Eqs. (51), (43), one can easily follow the phase changes in the resonance and specular TCs. The value of $X_r(\vartheta,\Omega)$ changes its sign when ϑ crosses the point $\vartheta_{\rm res}$. Correspondingly, the R_r experiences the phase shift $\sim \pi$ while the phase of R_0 changes by $\sim 2\pi$ (see insets in Fig. 2).

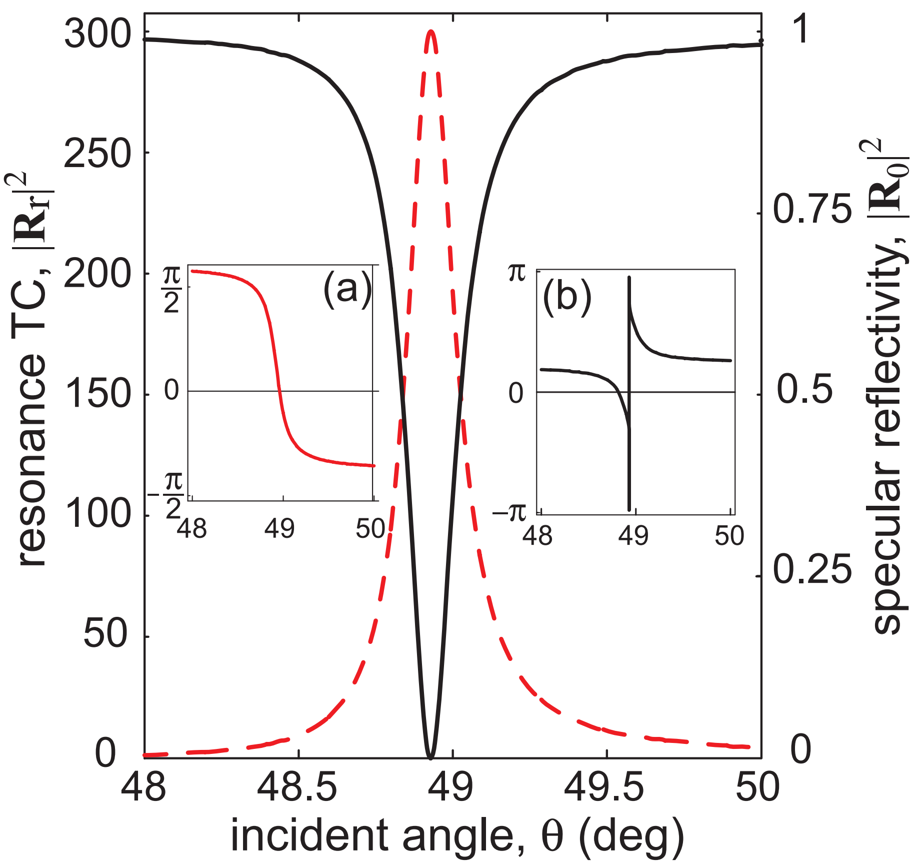

FIG. 2: (Color online) Numerical simulation of the total suppression of the specular reflection. Black solid and red dashed curves show the angular dependences of the specular and resonance TCs, respectively, for the forward resonance diffraction ($r = 1$) on the harmonic grating. These calculations were performed using Eqs. (34), (35) for the harmonic grating with pitch $L = 1$ mm and modulation magnitude $|f_r| = 3.65 \cdot 10^{-6}$. Other parameters used here are: $D = 2 \cdot 10^{-7}$ cm, $\lambda_{ab} = 2.5 \cdot 10^{-5}$ cm, $\nu = 10^{-7}$, $\varepsilon = 20$, and $(1 - \Omega^2) = 1.2 \cdot 10^{-5}$. The insets show the angular dependences of the phases of the resonance (a) and specular (b) TCs.

We also illustrated the effect of total suppression of the specular reflection by the distribution of the total magnetic field in the vacuum, Fig. 3. The interference pattern is seen for the non-resonant case, when the amplitudes of the incident and reflected waves practically coincide. Under the resonance condition, when the reflected wave is totally suppressed, the interference pattern in the far field disappears, while the near-field "torch" structure of the SJPW is clearly seen near the vacuum-layered superconductor interface.

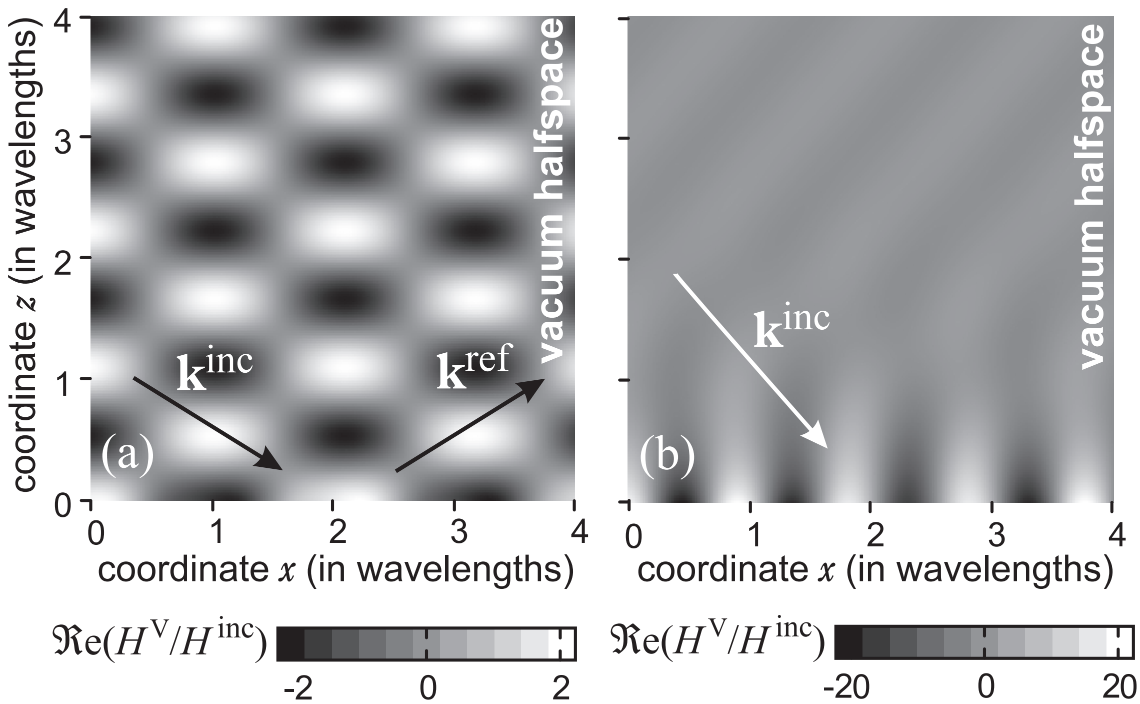

FIG. 3: The magnetic field distribution for the non-resonant case, $\theta = 60^o$, shown in (a), and for the resonant diffraction in diffraction order +1, $\theta = \theta_{\rm res} = 48.93^o$, shown in (b). Other parameters are the same as in Fig. 2.

Conclusion

In this paper, we present the detailed examination of the resonance features for the diffraction of THz radiation on periodically-modulated layered superconductors. The resonance is caused by the excitation of the SJPWs for definite combinations of the incident angle and frequency, and is analog to the widely studied *surface plasmon polariton resonance* in

the visible and near-infrared region. The analytical approach developed here allows us to predict strong resonance effects (total suppression of the specular reflection and total absorption) for specific combinations of the parameters. The simplest (in-plane) configuration for TM-polarized incident wave was examined here under single-resonance conditions (i.e., excitation of one running SJPW). This approach allows a similar study of the simultaneous excitation of two SJPWs (double resonance), as well as the examination of the so-called "conical diffraction mount" (out-of-plane diffraction). These items will be studied in the future. It seems interesting also to consider the resonance diffraction features for superconducting films of finite thickness. There, the effects of resonance enhancement of the transmissivity could exist.

The strongly selective interaction of SJPWs with the incident wave having a certain frequency and direction of propagation can be used for designing future THz detectors and filters. For instance, the simplest design of a THz detector could be built around a BiSrCuCaO sample fixed on a precisely rotated holder and attached by contacts to measure its resistance. When rotating the sample, the incident THz radiation would produce a surface wave at certain angles. This results in a strong enhancement of the absorption. Respectively, the sample temperature increases, thus, its resistance would increase.

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